



Lesson 7

RESONANCE AND FILTERS

ACMA Syllabus February 2024 Chapter 3.2

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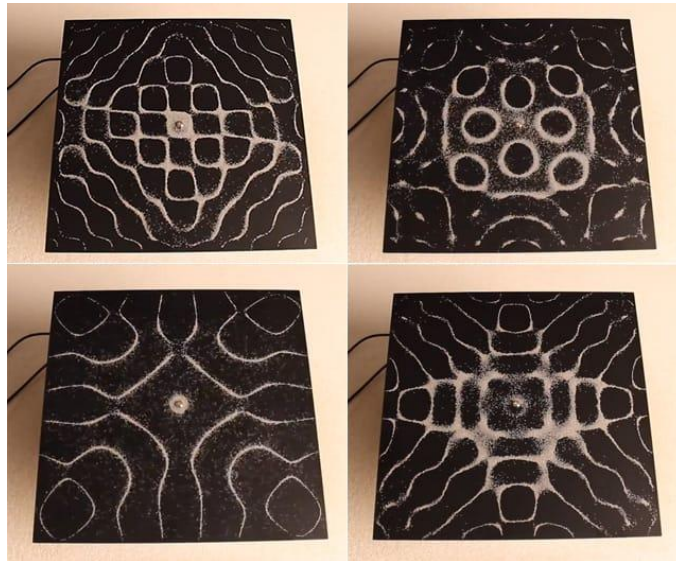
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Resonance

Resonance is a fundamental phenomenon in nature when a vibration matches its own natural frequency.



Resonance can be found in electronic circuits using capacitors and inductors.

The useful thing about LC circuits is that with any combination of the C value and the L value, the circuit will have a resonant frequency. At this resonant frequency, the circuits act differently than the components individually.

We saw in lesson 6, that capacitors and inductors have a resistance to AC called reactance. Also, RC and RL circuit have a calculatable charging times.

Reactance and Impedance

Every electronic component has DC resistance. Even the length of wire has resistance, however small.

Reactance

The component's resistance to AC signals is called reactance. Reactance resists current without dissipating power, unlike resistors.

Inductive reactance increases with frequency and inductance.

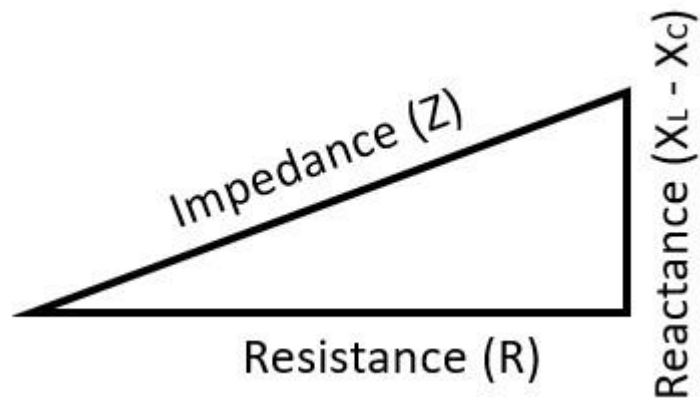
Capacitive reactance decreases with frequency and capacitance.

The reactance of a capacitor (X_C) or inductor (X_L) can be calculated depending on the frequency. Reactance was addressed in Lesson 6.

Impedance

Impedance is the combination of resistance and reactance and represents a combined total opposition.

In the sketch, DC resistance is shown in the X plane, and reactance is shown in the Y plane. Impedance is the vector joining these two.



Impedance Numbers

Impedance is written as $Z = R (+ \text{ or } -) j X$. You come across these numbers, as an example, when measuring the resonance of an antenna on a Smith Chart.

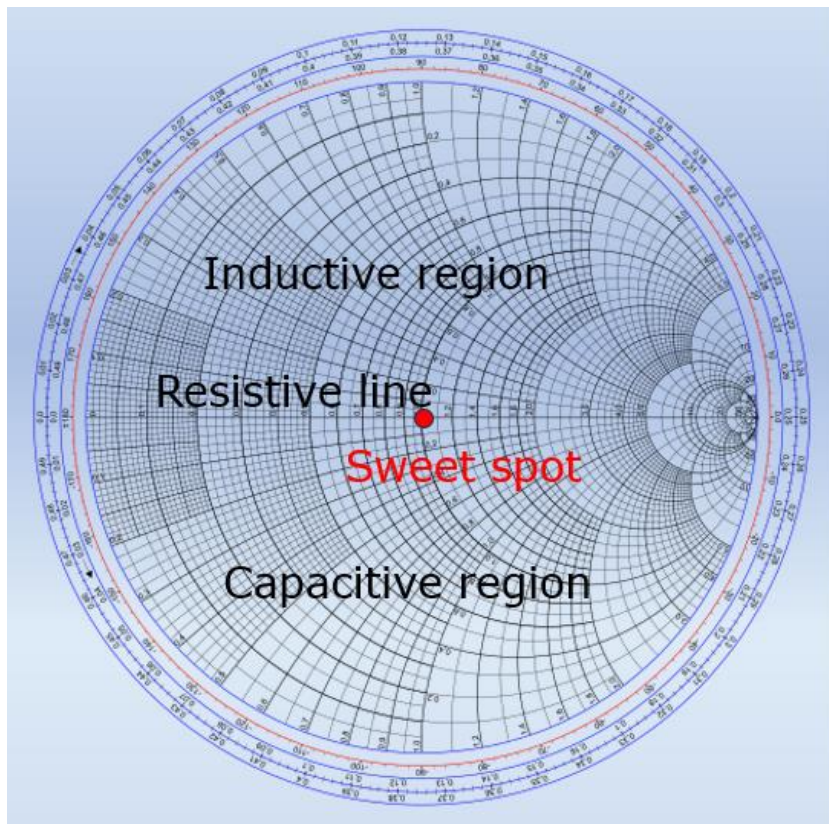
Example

$$Z = 32 + j 25 \Omega$$

32 Ω of DC resistance and a plus sign indicates an inductive reactance of 25 Ω .

$$Z = 43 - j64 \Omega$$

43 Ω of DC resistance and a minus sign indicates capacitive a reactance of 64 Ω .



Reactance at Resonance (IMPORTANT)

At the resonant frequency of a series or parallel LC circuit, $X_L = X_C$ and the impedance (Z) is reduced to R. X_L and X_C cancel because inductive reactance is positive and capacitive reactance is negative.

Example

$$Z = 13 + j56\Omega$$

And

$$Z = 13 - j56\Omega$$

The $j56\Omega$ cancel leaving

$$Z = 13\Omega.$$

In a **series** tuned circuit, at resonance, there is a **low** resistance path for the signal through the circuit.

In a **parallel** tuned circuit, at resonance, there is a **high** resistance path for the signal through the circuit. X_L and X_C are equal. The charge and discharge times for the inductor and capacitor are in sync and the circuit will **oscillate**.

Series Resonant LC Circuit

In a series LC circuit, the energy goes through one component then the next. Any signal below the resonant frequency will be blocked by the capacitor. Signals above the resonant frequency will be blocked by the inductor. Signals at the resonant frequency are not blocked by the capacitor or the inductor.

At resonance in the series tuned circuit, the reactance cancels each other leaving only resistance in the circuit. So, at resonance, this circuit has a low impedance to the alternating signal.

Remember: At resonance X_L and X_C cancel leaving R



Series Circuit

These LC circuits are an essential building block in communications equipment. They can act as filters, tuning circuit for a radio, or the basis of an oscillator.

- High impedance below the resonant frequency (Blocked by the capacitor)
- Low Impedance at the resonant frequency (Signal passes through the circuit)
- High impedance above the resonant frequency (Blocked by the inductor)

Parallel LC Circuit

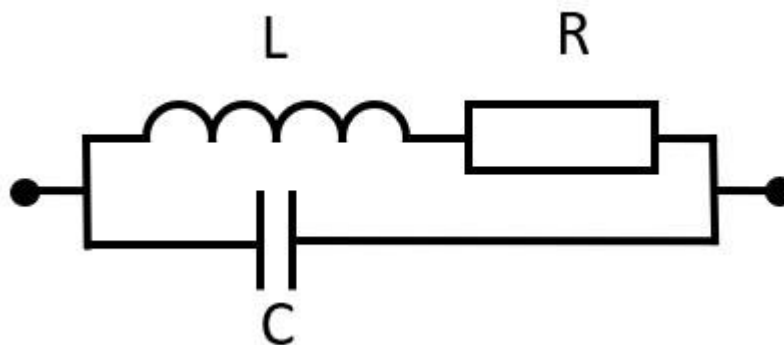
A parallel LC circuit, also called a resonant circuit, tank circuit, or tuned circuit. The circuit can act as an electrical resonator, an electrical analogue of a tuning fork at the circuit's resonant frequency.

A parallel LC circuit will block any signal that is operating on the resonant frequency. In a parallel LC circuit at resonance, the time taken for the capacitor to charge and discharge, is the same time taken for the inductor to charge and discharge.

The energy in the circuit will charge the capacitor and, as the capacitor discharges, the energy will charge the inductor. As the inductor discharges, the energy recharges the capacitor. So, at the resonant frequency, the parallel LC circuit will oscillate.

The combined reactance is high increasing the impedance. So, at resonance, the parallel tuned circuit has a high impedance to the alternating signal.

A signal at any other frequency will pass through the parallel LC circuit.



Parallel Circuit

- Low impedance below the resonant frequency (Path through the inductor)
- High Impedance at the resonant frequency (Signal is blocked)
- Low impedance above the resonant frequency (Path through the capacitor)

Frequency Calculation

The resonant frequency for any combination of capacitor and inductor can be calculated using the formula below.

$$f = \frac{1}{2\pi\sqrt{L \times C}}$$

f = frequency in hertz (Hz)

C = capacitance in Farads (F)

L = inductance in Henries (L)

π = 3.141 (Pi)

Example: What is the resonant frequency of a capacitor of 1 uF and an inductor of 1 uH

$$\begin{aligned}
 f &= \frac{1}{2 \times \pi \times \sqrt{L \times C}} \\
 &= \frac{1}{2 \times 3.141 \times \sqrt{0.000001 \times 0.000001}} \\
 &= \frac{1}{6.282 \times 0.000001} \\
 &= \frac{1}{0.000006282} \\
 &= 159,184 \text{ Hz or } 159.184\text{kHz}
 \end{aligned}$$

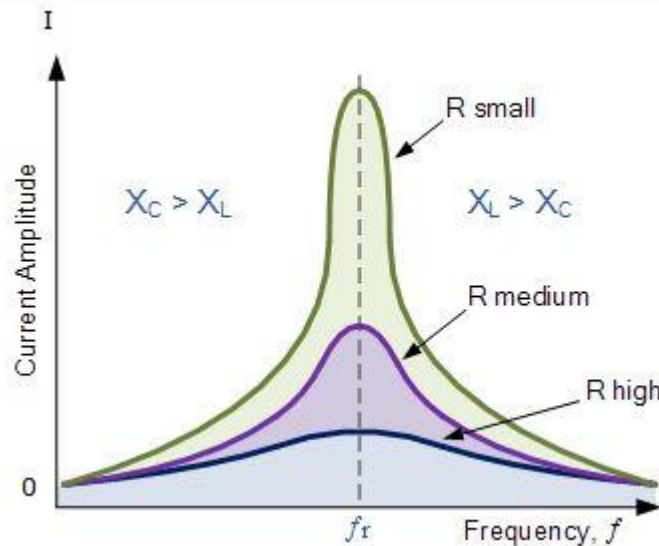
Q Factor

The ratio of the reactance to the resistance, in an operational circuit, is called the Q (Quality Factor) of the circuit. If the current in the circuit is plotted against frequency, either side of the resonant frequency, the result will be a Q plot resembling a bell curve. The base of the bell and the height of the bell indicate the bandwidth of the circuit.

A sample Q plot of a series tuned circuit is shown.

Below resonant frequency, the capacitive reactance X_C is predominant.

Above resonant frequency, the inductive reactance X_L is predominant.



Series Circuit Q

The Q of a series circuit, is the ratio of the reactance, at resonance X_C or X_L , over the resistance.

$$Q = \frac{X}{R_S} \quad \text{or} \quad Q = \frac{2\pi f L}{R_S}$$

Q = Quality factor

X = Reactance of capacitor or inductor (Calculated from $2 \pi f L$)

R_S = Series resistance in Ohms.

Example Series circuit XL and XC 350 Ω and a resistance of 10 Ω

$$Q = \frac{X}{RS}$$
$$= \frac{350}{10}$$
$$Q = 35$$

Typical Series Circuit Q Factor Values

Low Q (< 10): Not useful for resonance-based tasks. Very wide bandwidth and are highly dampened.

Standard Electronics (10 – 100): This is the typical range for mass-produced components like inductors and common RLC tuning circuits.

High Q (> 100): Found in highly selective equipment like radio receivers and precision filters. Values above 1000 are difficult to achieve with standard coils.

Extreme High Q (1,000+): Tuning forks have a Q of around 1,000, while atomic clocks or superconducting cavities can reach values of 10¹¹

Parallel Circuit Q

Were R, L and C are in parallel, the lower the parallel resistance, the more effect it will be in damping the circuit and lower the Q. This is useful in filter design to determine the bandwidth. The formula to calculate Q is shown below.

$$Q = \frac{Rp}{X} \text{ or } Q = \frac{Rp}{2 \times \pi \times f \times L}$$

Q = Quality factor

X = Reactance of capacitor or inductor

R_P = Parallel resistance in Ohms.

Example Parallel circuit XL and XC 350 Ω and a resistance of 1000 Ω

$$Q = \frac{RP}{X}$$
$$= \frac{1000}{350}$$
$$Q = 2.8$$

Typical Parallel Circuit Q Factor Values

Low Q (0.5 – 5): Ideal for broadband filters where you want to pass a wide range of frequencies or for critically damped systems.

Moderate Q (10 – 100): The standard for most consumer electronics and basic tuning circuits.

High Q (100 – 300+): Necessary for highly selective RF filters and oscillators.

Ultra-High Q (1,000+): Rare in simple RLC circuits but found in crystal oscillators.

Bandwidth

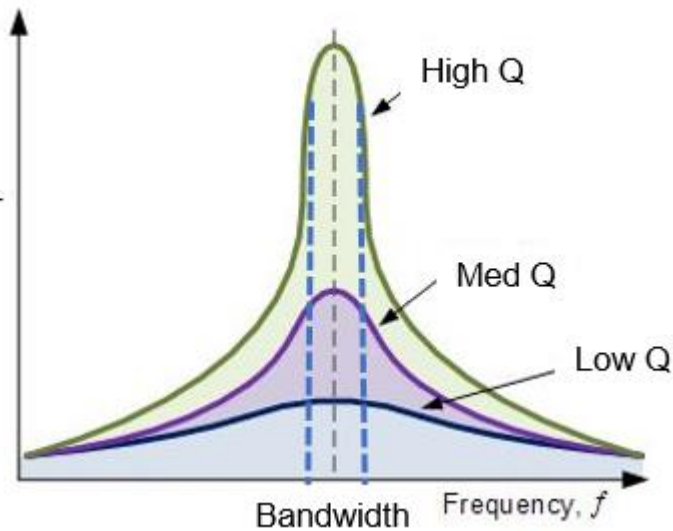
Bandwidth (BW) of a circuit is related to the frequency and Q of the circuit. The higher the Q, the narrower the bandwidth. Bandwidth can be calculated.

$$BW = \frac{f}{Q}$$

BW = hertz (Hz)

F = hertz (Hz)

Q = quality factor.



Example: What is the bandwidth of a 14 MHz circuit with a Q of 35?

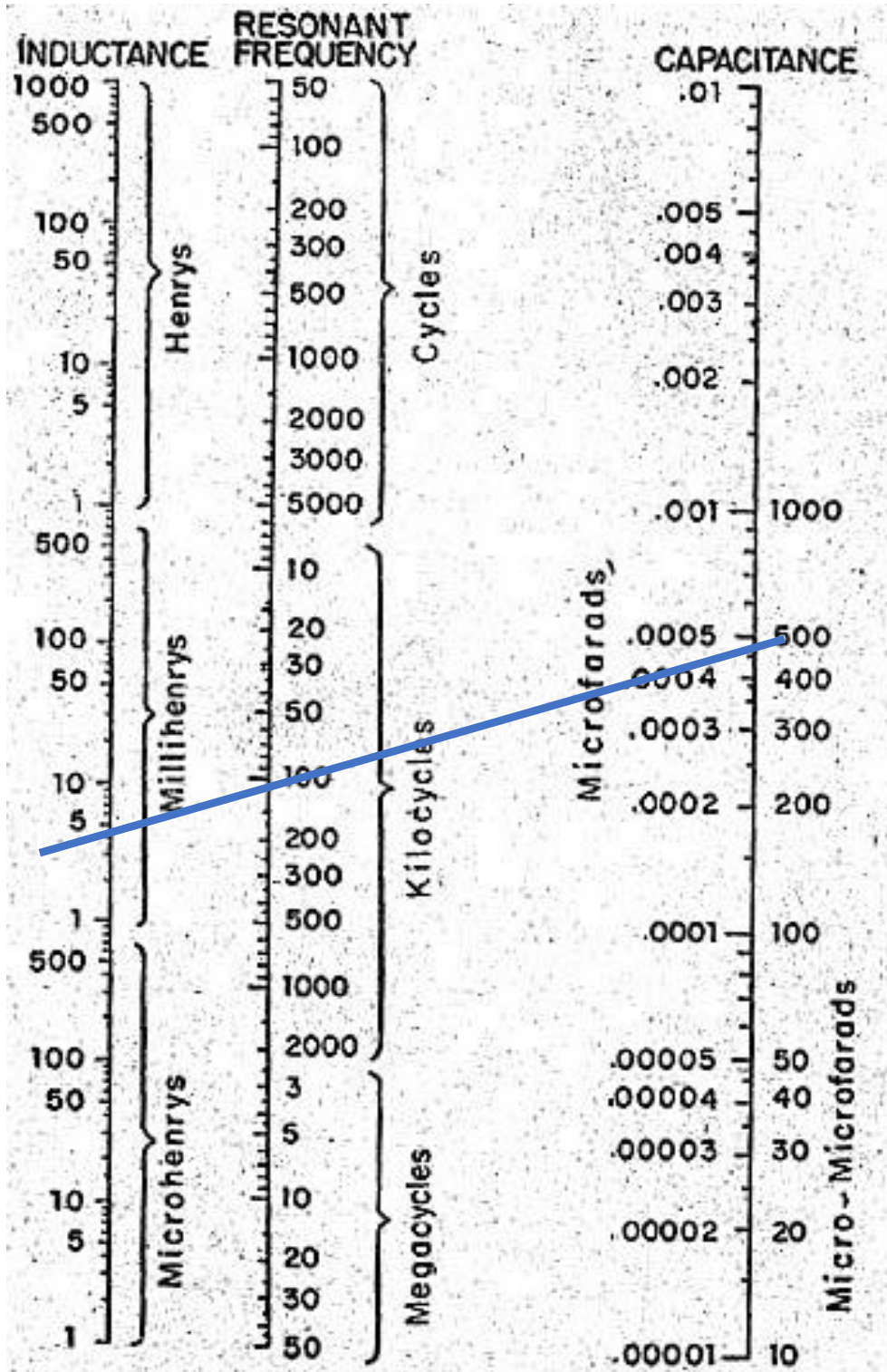
$$\begin{aligned} BW &= f / Q \\ &= 14000000 / 35 \\ &= 400,000 \text{ Hz} \end{aligned}$$

Hint: As Q increases, the BW gets smaller.

Reference Table

I found the following table in Coyne's Television and Radio Handbook P183. This gives a good starting point to match L and C for a particular frequency.

Draw a line from the desired capacitance to the desired inductance and the line will pass through the estimated resonant frequency. In the example shown, a 5 mH inductor and a 500 nF capacitor would resonate at approximately 100 kHz.



Filters

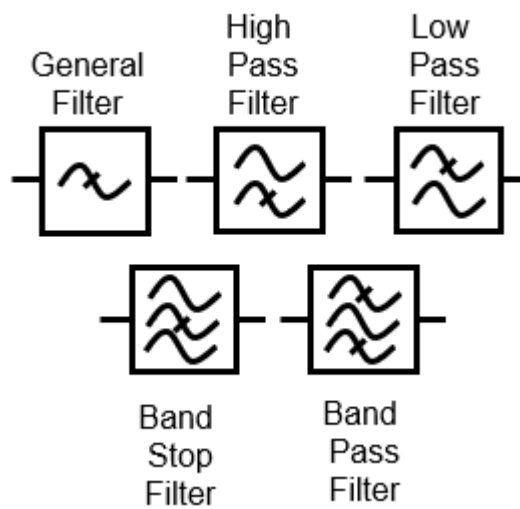
Electronic filters remove unwanted frequency components from the signal or enhance the wanted signal.

Filters fall into four basic categories.

- Low pass (LPF)
- High pass (HPF)
- Band pass. (BPF)
- Band stop. (BSF)

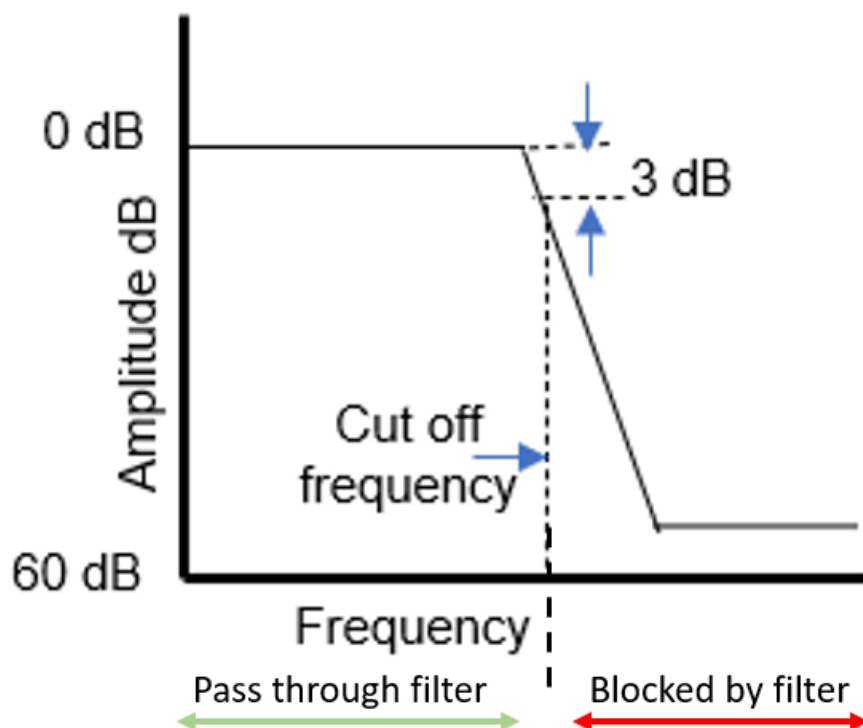
Filters can be constructed with active and or passive components. Only passive components will be used in the following examples.

Filter Symbols



Low-pass filters

A low-pass filter (LPF) only passes signals below its cutoff frequency while attenuating all signals above the cut off frequency.



The frequency cut off point is when the signal drops 3dB. 3dB down means the voltage is reduced to approximately 70.7% of its original value. The reason 3dB is chosen, is because the drop off point is not clear like the drawing.

Choke

A choke is a simple low pass filter. The inductor will pass signals until the Inductive reactance rises high enough to block the signals.



Example: At what frequency would a 100mH choke reach a reactance 1kΩ?

$$X_L = 2 \times \pi \times f \times L$$

$$f = \frac{X_L}{2 \times \pi \times L}$$

$$f = \frac{1000}{2 \times 3.141 \times 0.1}$$

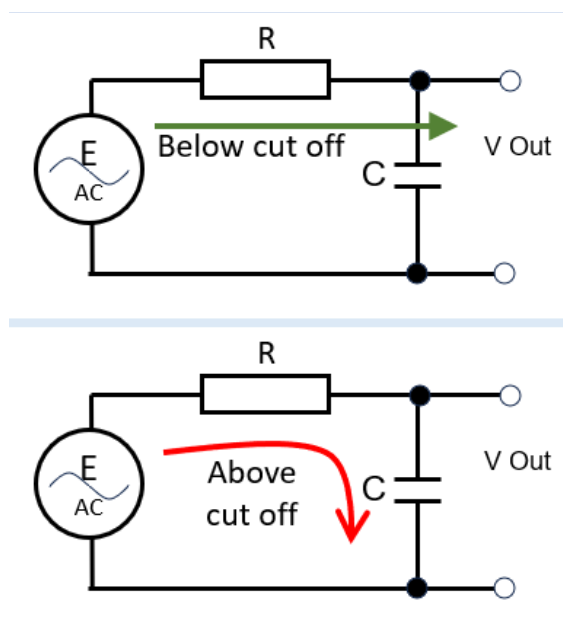
$$f = \frac{1000}{0.6282}$$

$$f = 1591.8 \text{ Hz}$$

Hints: X_L is inductive reactance and the formula was covered in Lesson 7 resonance.

RC Filter

The RC low pass filter will allow low frequency signals to pass until the capacitive reactance (X_C) drops low enough to pass signals.



Example: What size capacitor is needed to prevent signals above 1MHz passing to V Out?
Assume XL = 10 Ohms.

$$X_C = \frac{1}{2\pi f C}$$

$$X_C \times 2 \times 3.141 \times 1000000 = \frac{1}{C}$$

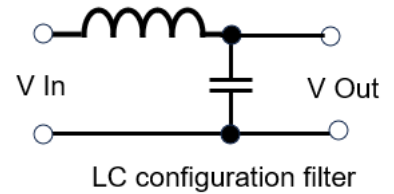
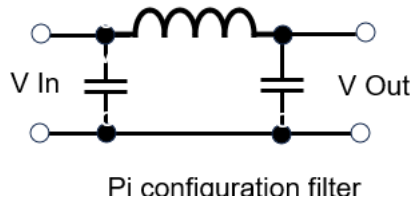
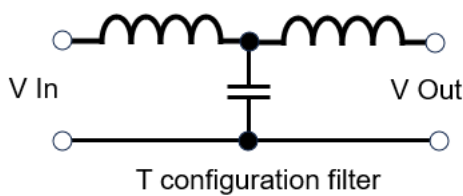
$$10 \times 2 \times 3.141 \times 1000000 = \frac{1}{C}$$

$$62,820,000 = \frac{1}{C}$$

$$C = 15.9 \text{ nF}$$

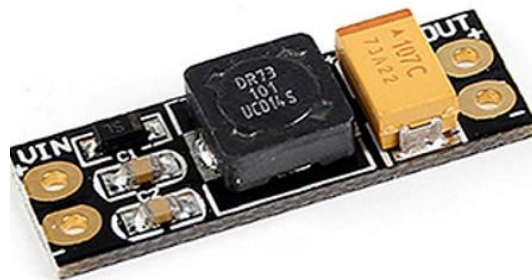
LC Low Pass Circuits

Low-pass LC circuits are passive filters composed of inductors (L) and capacitors (C) that allow low-frequency signals to pass while attenuating higher frequency signals. The cut off is -3dB down.



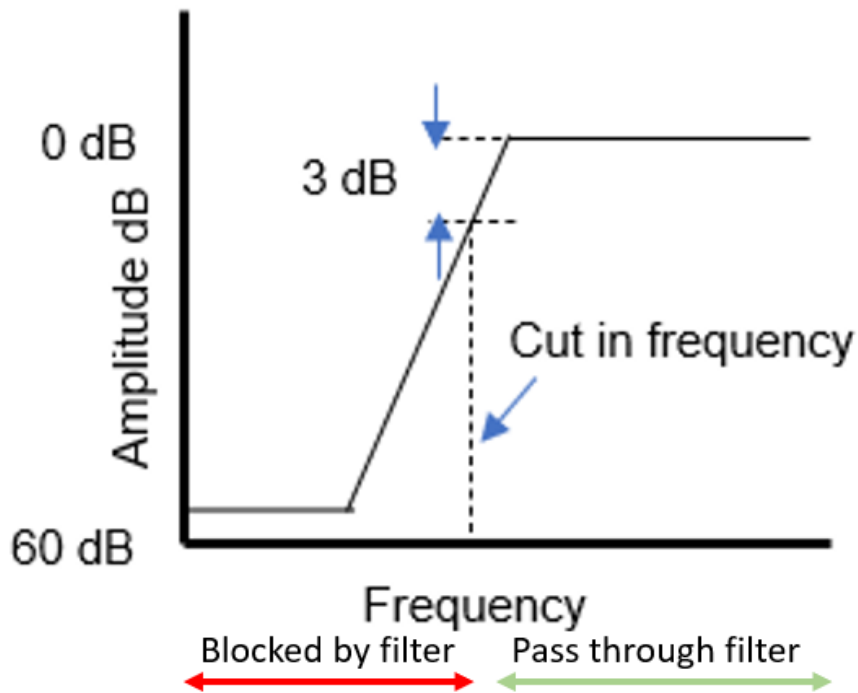
Below cut off frequencies, the inductors let the signal pass.
Above cut off frequencies the capacitors pass the signal to ground.
The key to setting the cut off frequency is the capacitive reactance XC.

Hint: Study the three configurations.



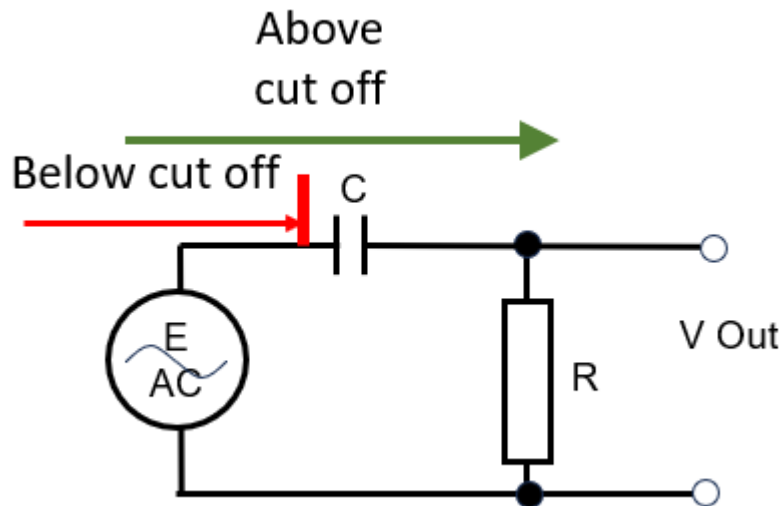
High-pass filters

A high-pass filter (HPF) only passes signals above its cutoff frequency and attenuates all signals below.



RC Filter

The RC high pass filter will allow high frequency signals to pass until the capacitive reactance (X_C) rises high enough to block signals.



Example: What size capacitor is needed to prevent signals below 1kHz passing to V Out?
Assume XL = 10 Ohms.

$$X_C = \frac{1}{2\pi f C}$$

$$X_C \times 2 \times 3.141 \times 1000 = \frac{1}{C}$$

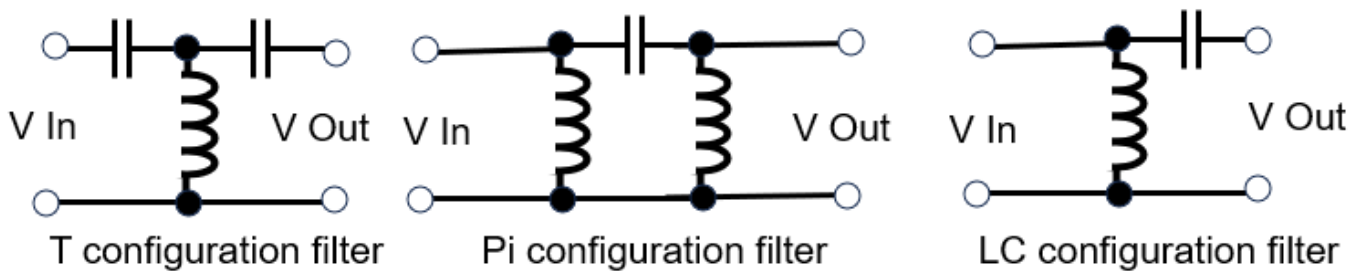
$$10 \times 2 \times 3.141 \times 1000 = \frac{1}{C}$$

$$62,820 = \frac{1}{C}$$

$$C = 15.9\mu\text{F}$$

LC High Pass Circuits

High-pass LC circuits are passive filters composed of inductors (L) and capacitors (C) that allow high-frequency signals to pass while attenuating lower frequency signals. The cut in is -3dB down.



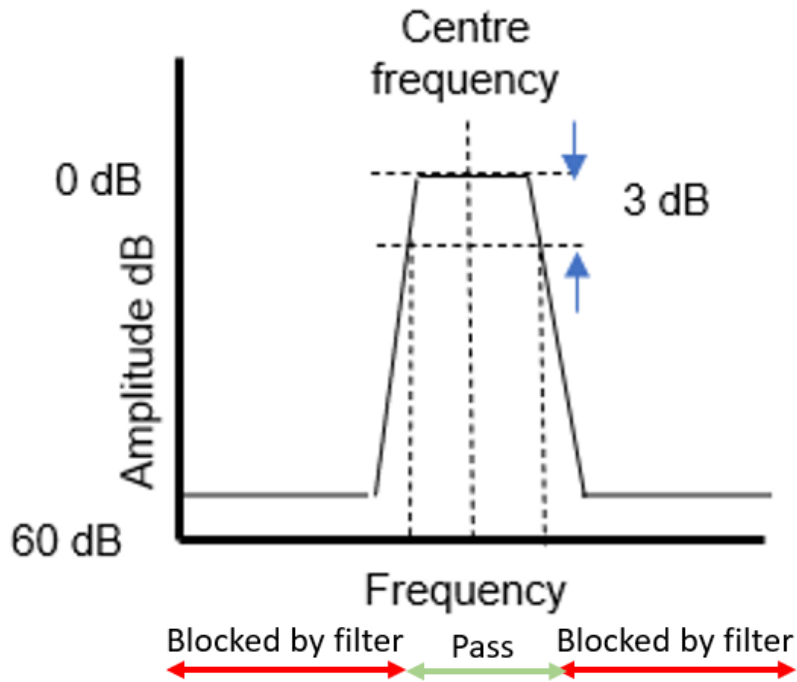
Below cut in frequencies, the capacitors block the signal.
Above cut in frequencies the capacitors pass the signal.
The key to setting the cut off frequency is the capacitive reactance XC.

Hint: Study the three configurations.



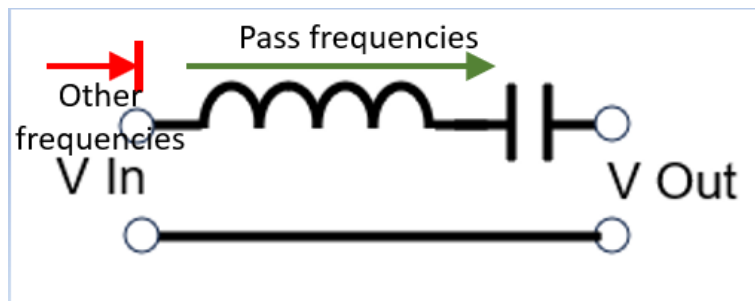
Band-pass filters

A band pass filter (BPF) allows signals within a selected range of frequencies to pass and any frequencies outside this band are attenuated.

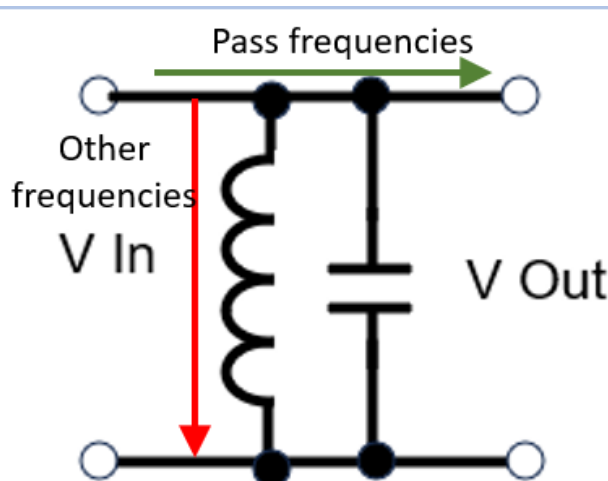


Configurations

Series: The series LC circuit will only allow the resonant frequency to pass through and block other frequencies.



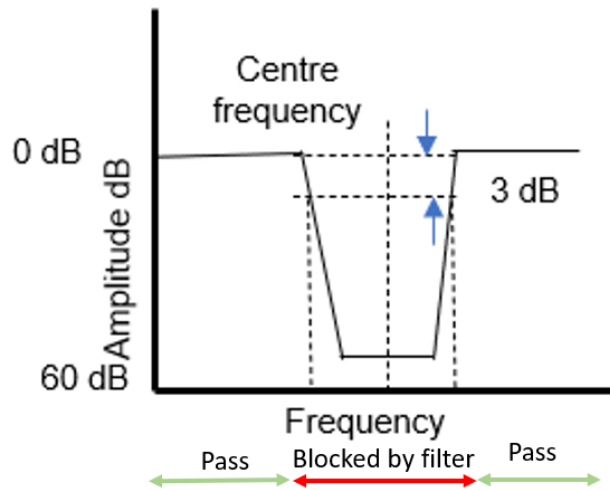
Parallel: The parallel LC circuit will allow all frequencies, except the resonant frequencies to pass to earth. The parallel tuned circuit has a high impedance at resonance but a low impedance at other frequencies.





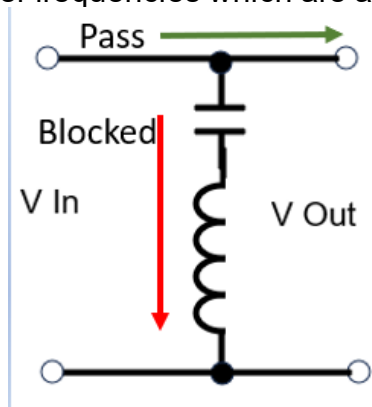
Band-stop filters.

A band stop filter (BSF) attenuates signals within a selected range of frequencies and allows the frequencies outside this band to pass.

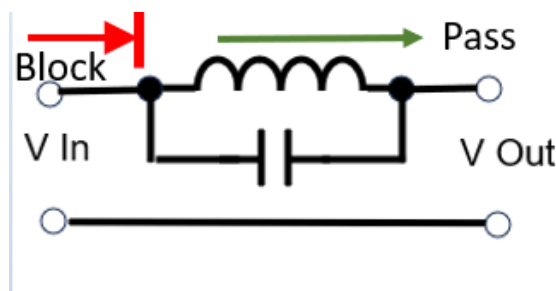


Configurations

Series - The series LC tuned circuit will pass the unwanted frequency, this is the circuits resonant frequency to earth and blocking other frequencies which are allowed to pass through.



Parallel - The parallel LC tuned circuit will block the frequencies that are the circuit's resonant frequencies and pass others.

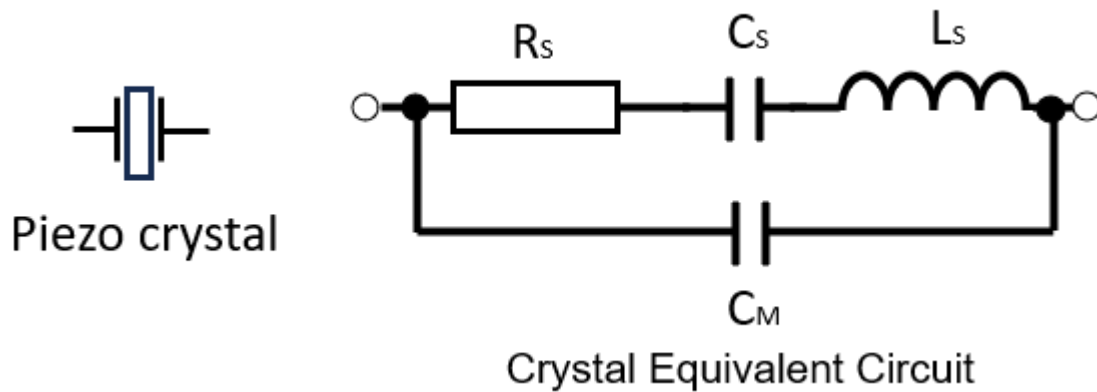


Quartz crystal Filters

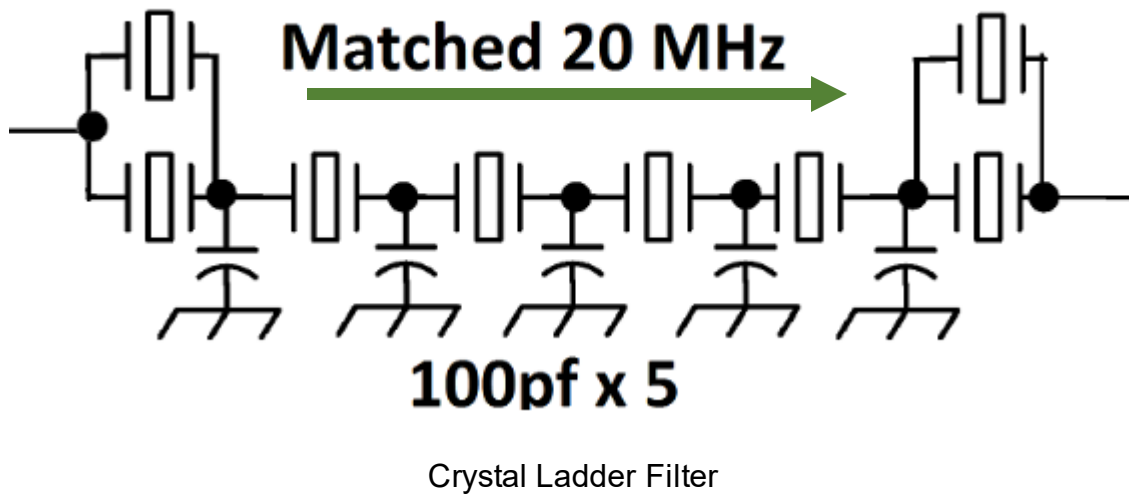
A crystal is a piezoelectric device that uses mechanical resonance to achieve a very precise frequency output.

The quartz crystal generates an electrical signal when subjected to mechanical stress or, when a voltage is applied, the crystal oscillates

The most common type of crystal material is quartz, but different piezoelectric materials may be used.



When the crystal is not vibrating, it is equivalent to the capacitance C_m .
 When the crystal vibrates, it acts like a series tuned circuit (R_s - C_s - L_s).



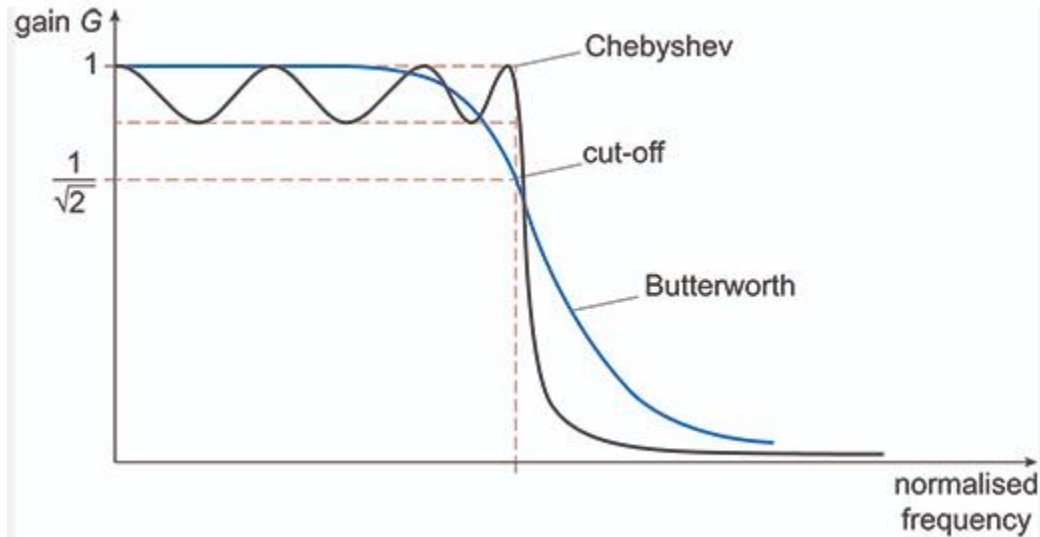
Crystal ladder filters use multiple quartz crystals to create sharp, narrow-band filters for high-performance radio receivers (especially SSB/CW).

Each crystal acts as a resonator. Adding more crystals increases the filter's order, resulting in steeper roll-off (steeper sides on the passband) and better rejection of unwanted adjacent signals

Filter Variations

Butterworth Filter

The Butterworth filter is an analogue filter design. This filter produces the best output response without ripple in the pass band or the stop band region.

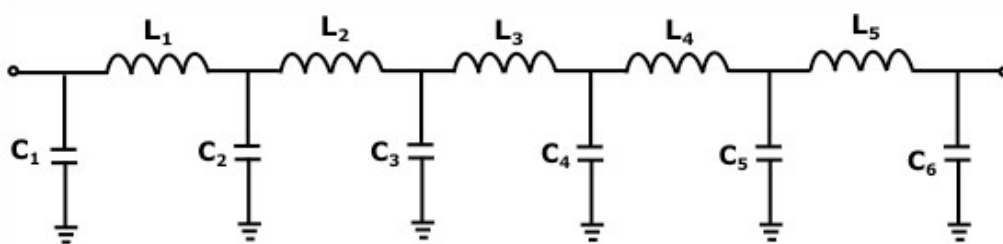


Chebyshev Filter

The Chebyshev filter is popular in RF applications - using inductor and capacitor, LC combinations to provide a fast transition from passband to stopband. This transition comes at the cost of in-band ripple, and this may not make it suitable for all applications.

Component Values

Both Butterworth and Chebyshev filters use the same ladder topology (inductors and capacitors), the specific calculated values for the components differ significantly to achieve their respective responses.



Digital Filters

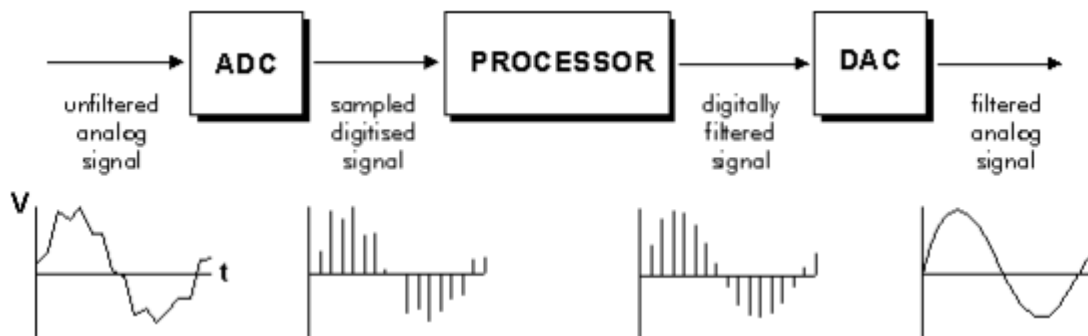
An analogue filter uses electronic circuits made from components such as resistors, capacitors, and inductors to produce the required filtering effect. There are well-established standard techniques for designing an analog filter circuit for a given requirement.

A digital filter uses a digital processor to perform numerical calculations on sampled values of the signal. The processor may be a general-purpose computer such as a PC, or a specialised DSP (Digital Signal Processor) chip.

Input the signal to the Analogue to Digital Converter (ADC). The output of the ADC is digital data which can be manipulated according to the desired effect by algorithms in the processor.

The output of the processor goes to the Digital to Analogue Converter (DAC) which turns the digital signals back to analogue signals.

The digital data can be manipulated according to the desired effect by algorithms in the processor.



Go to Lesson 7 questions.

